T-79.1001 **Spring 2006**
Introduction to Theoretical Computer Science T
Tutorial 8, 28–29 March
Problems

Homework problems:

1. Show, using the pumping lemma for regular languages, that the language consisting of even-length palindromes,

$$\{ww^R \mid w \in \{a,b\}^*\}$$

is not regular.

2. Show, using the pumping lemma for context-free languages, that the language

$$\{ww \mid w \in \{a,b\}^*\}$$

is not context-free. (*Hint:* Consider strings of the form $a^nb^nca^nb^n$.)

3. Let A and B be countably infinite sets such that $A \cap B = \emptyset$. Show that then also the set $A \cup B$ is countably infinite. (*Extra question:* Show that the claim holds even without the assumption $A \cap B = \emptyset$.)

Demonstration problems:

4. Prove that the class of context-free languages is closed under unions, concatenations, and the Kleene star operation, i.e. if the languages $L_1, L_2 \subseteq \Sigma^*$ are context-free, then so are the languages $L_1 \cup L_2$, L_1L_2 and L_1^*.

5. Prove that the class of context-free languages is not closed under intersections and complements. (*Hint:* Represent the language $\{a^kb^kc^k \mid k \geq 0\}$ as the intersection of two context-free languages.)

6. Prove that the Cartesian product $\mathbb{N} \times \mathbb{N}$ is countably infinite. (*Hint:* Think of the pairs $(m,n) \in \mathbb{N} \times \mathbb{N}$ as embedded in the Euclidean (x,y) plane $\mathbb{R}^2$. Enumerate the pairs by diagonals parallel to the line $y = -x$.) Conclude from this result that also the set $\mathbb{Q}$ of rational numbers is countably infinite.