T-79.1001/1002 **Spring 2006**
Introduction to Theoretical Computer Science T/Y
Tutorial 6, 28–29 February
Problems

There are no lectures on the course in week 9 (27 Feb – 3 Mar), but the tutorial sessions are arranged as usual. Week 10 (6–10 Mar) is exam week, and this course has neither lectures nor tutorials on that week. The final exam for the short version (T-79.1002, 2 cr) of the course takes place on Thu 9 Mar, and covers the topics discussed on the course so far (Lectures 1–6). You must register for the exam via the TOPI system by Mon 6 Mar 8 a.m. Also, all the compulsory Regis problems must have been solved by the time of the exam. The exam for the long version of the course (T-79.1001, 4 cr) takes place on Fri 19 May. Participants in the long version of the course should NOT go to the exam on 9 Mar. (Exception: the repeat exam for the participants of T-79.1001 from Autumn 2005 takes place on that date.)

Homework problems:

1. Consider the following grammar generating a certain type of list structures:

$$S \to (S) \mid S, S \mid a.$$

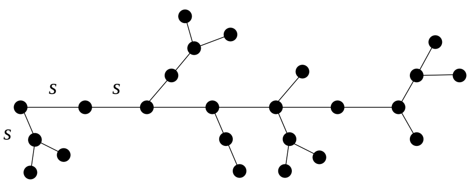
   (a) Based on the above grammar, give a leftmost and rightmost derivation and a parse tree for the sentence "$(a, (a))$".
   (b) Prove that the grammar is ambiguous.
   (c) Design an unambiguous grammar generating the same language.

2. Construct context-free grammars for the following languages:

   (a) $\{ucv \mid u, v \in \{a, b\}^* \text{ and } |u| = |v|\}$,
   (b) $\{a^m b^n \mid m > n\}$,
   (c) $\{a^m b^n \mid m \neq n\}$.

   Additionally, give a derivation for the string $abcab$ using your first grammar and another for $aaabb$ using your second grammar. For the third grammar, note that $m \neq n$ if and only if $m > n$ or $m < n$.

3. A *fern* consists of a stem and a number of subferns rooted on the left and right sides of the stem. For instance, the following structure is a fern:



   A fern structure can be described by a string where each unit of the stem is denoted by a letter s, and each subfern is described by a similar string in parentheses, located at the point where the subfern is rooted, and prefixed by l or r depending on whether the subfern occurs on the left or right side of the main stem, respectively. At most one subfern can be rooted to the left and to the right at each point, and each subfern must contain at least one stem unit. For instance, the string representation corresponding to the above example would be:

$$r(sl(s)r(s))ssl(ssl(s)r(s))sr(ss)sl(s)r(sl(s)r(s))ssl(sr(s)s)r(s).$$

   Design a context-free grammar describing the structure of such fern strings.

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Demonstration problems:

4. Design a context-free grammar for the language $\{w \in \{a,b\}^* \mid w \text{ contains equally many } a\text{'s and } b\text{'s}\}$.

5. (a) Prove that the following context-free grammar is ambiguous:

$$\begin{aligned} S &\rightarrow \textbf{if } b \textbf{ then } S \\ S &\rightarrow \textbf{if } b \textbf{ then } S \textbf{ else } S \\ S &\rightarrow s. \end{aligned}$$

   (b) Design an unambiguous grammar that is equivalent to the grammar in item (a), i.e. that generates the same language. (*Hint:* Introduce new nonterminals B and U that generate, respectively, only "balanced" and "unbalanced" **if-then-else-**sequences.)

6. Design a recursive-descent (top-down) parser for the grammar from Problem 5 of the previous tutorial.