T-79.1001/1002 **Spring 2006**
Introduction to Theoretical Computer Science T/Y
Tutorial 5, 21–22 February
Problems

Homework problems:

1. Consider the following context-free grammars:

$$\text{(a)}\quad \begin{array}{rcl} A & \to & aAcc \mid B \\ B & \to & bBc \mid \varepsilon \end{array} \qquad\qquad \text{(b)}\quad S \to +S- \mid SS \mid \varepsilon$$

Give a derivation for the sentence $abccc$ according to grammar (a), and a derivation for the sentence $++-+--+-$ according to grammar (b). Describe the language generated by each grammar verbally as simply as you can.

2. A *palindrome* is a string w such that $w = w^R$. (E.g. "MADAMIMADAM", "ABLEWASIEREISAWELBA," cf. `http://www.palindromelist.com/`.) Consider the set of palindromes over the alphabet $\{a, b\}$:

$$\text{PAL} = \{w \in \{a, b\}^* \mid w = w^R\}.$$

Design a context-free grammar generating the language. (*Hint:* Note that a string $w \in \text{PAL}$, if and only if it is of the form $w = uXu^R$, where $X = a$, b or ε.)

3. Design right-linear grammars for the following languages:

   (a) $\{w \in \{a, b\}^* \mid w$ does not contain abb as a substring$\}$;

   (b) $\{w \in \{0, 1\}^* \mid w$ contains an even number of 0's and an odd number of 1's$\}$.

   (Cf. Problem 4/3.)

Demonstration problems:

4. Prove that the class of context-free languages is closed under unions, concatenations, and the Kleene star operation, i.e. if the languages $L_1, L_2 \subseteq \Sigma^*$ are context-free, then so are the languages $L_1 \cup L_2$, L_1L_2 and L_1^*.

5. Design a context-free grammar describing the syntax of simple "programs" of the following form: a program consists of nested `for` loops, compound statements enclosed by `begin-end` pairs and elementary operations `a`. Thus, a "program" in this language looks something like this:

```
a;
for 3 times do
begin
   for 5 times do a;
   a; a
end.
```

For simplicity, you may assume that the loop counters are always integer constants in the range 0, … , 9.

PLEASE TURN OVER

6. In the modern WWW page description language XML, designers can construct their own "data type definitions" (abbr. DTD), which are essentially context free grammars describing the structure of the text or other data displayed on the page. Acquaint yourself with the notation used in this XML/DTD description language (from e.g. `http://www.rpbourret.com/xml/xmldtd.htm`), and give a context-free grammar corresponding to the following XML/DTD description:

```
<!DOCTYPE Book [
   <!ELEMENT Book (Title, Chapter+)>
   <!ATTLIST Book Author CDATA #REQUIRED>
   <!ELEMENT Title (#PCDATA)>
   <!ELEMENT Chapter (#PCDATA)>
   <!ATTLIST Chapter id ID #REQUIRED>
]>
```