T–79.1001 **Autumn 2006**
Introduction to Theoretical Computer Science T
Tutorial 8, 14 to 15 November
Problems

Homework problems:

1. Design pushdown automata recognising the following languages:
   (a) $\{w \in \{a,b\}^* \mid w = w^R\}$;
   (b) The language generated by grammar
   $$S \rightarrow (S) \mid S, S \mid a$$
   .

2. Show, using the pumping lemma for context-free languages, that the language
   $$\{ww \mid w \in \{a,b\}^*\}$$
   is not context-free. (*Hint:* Consider strings of the form $a^n b^n a^n b^n$.)

3. Design a Turing machine that recognises ("decides") the language
   $$\{1^n 0 1^n \mid n \geq 0\}.$$
   Show the computation sequences ("runs") of your machine on inputs 11011 and 1011.

Demonstration problems:

4. Prove that the class of context-free languages is not closed under intersections and complements. (*Hint:* Represent the language $\{a^k b^k c^k \mid k \geq 0\}$ as the intersection of two context-free languages.)

5. Design a pushdown automaton corresponding to the grammar $G = (V, \Sigma, P, S)$, where
   $$\begin{aligned} V &= \{S, (,), ^*, \cup, \emptyset, a, b\} \\ \Sigma &= \{(,), ^*, \cup, \emptyset, a, b\} \\ P &= \{S \rightarrow (SS), S \rightarrow S^*, S \rightarrow (S \cup S), \\ &\quad S \rightarrow \emptyset, S \rightarrow a, S \rightarrow b\} \end{aligned}$$

6. Design a grammar corresponding to the pushdown automaton $M = (Q, \Sigma, \Gamma, \Delta, s, F)$, where
   $$\begin{aligned} Q =& \{s, q, f\},\ \Sigma = \{a, b\},\ \Gamma = \{a, b, c\},\ F = \{f\}, \\ \Delta =& \{ \big((s,e,e),(q,c)\big), \big((q,a,c),(q,ac)\big), \big((q,a,a),(q,aa)\big) \\ & \big((q,a,b),(q,e)\big), \big((q,b,c),(q,bc)\big), \big((q,b,b),(q,bb)\big) \\ & \big((q,b,a),(q,e)\big), \big((q,e,c),(f,e)\big) \} \end{aligned}$$