Homework problems:

1. Design a Turing machine that recognises (“decides”) the language
   \( \{1^n01^n \mid n \geq 0\} \).
   Show the computation sequences (“runs”) of your machine on inputs 11011 and 1011.

2. Design a two-tape nondeterministic Turing machine that recognises the language
   \( \{ww \mid w \in \{a, b\}^*\} \).

3. Design a three-tape Turing machine ADD that functions as follows. The machine
   gets as input on tapes 1 and 2 two binary numbers written in reverse, i.e. with
   their least significant bits first. It then computes on tape 3 the sum of the two
   given numbers in the same notation. For simplicity, you may assume that the
   input numbers are of the same length, i.e. that the possibly shorter one is padded
   with leading zeros. Thus, for instance, the calculation \(7 + 11 = 18\) is represented
   as:

   \[
   \begin{array}{c}
   1110 \\
   1101 \\
   01001
   \end{array}
   \]

Demonstration problems:

4. Show that pushdown automata with two stacks (rather than just one as permitted
   by the standard definition) would be capable of recognising exactly the same
   languages as Turing machines.

5. Extend the notion of a Turing machine by providing the possibility of a two-way
   infinite tape. Show that nevertheless such machines recognise exactly the same
   languages as the standard machines whose tape is only one-way infinite.

6. Show that Turing machines whose tape alphabet contains at most two symbols
   in addition to the input symbols are capable of recognising exactly the same
   languages as the standard machines.