T-79.1001 **Autumn 2005**
Introduction to Theoretical Computer Science T
Tutorial 8, 15–16 November
Problems

Homework problems:

1. Show, using the pumping lemma for regular languages, that the language consisting of even-length palindromes,

$$\{ww^R \mid w \in \{a,b\}^*\}$$

is not regular.

2. Show, using the pumping lemma for context-free languages, that the language

$$\{ww \mid w \in \{a,b\}^*\}$$

is not context-free. (*Hint:* Consider strings of the form $a^n b^n c a^n b^n$.)

3. Prove that for any two countable sets A_1 and A_2, their union $A_1 \cup A_2$ is also countable. Deduce from this by induction that the same holds for the union of n countable sets $A_1 \cup A_2 \cup \ldots A_n$, for any $n \geq 2$. (*Extra question:* Does the claim still hold if the number of sets to be combined is countably infinite, e.g. $A = A_1 \cup A_2 \cup \ldots$, where each A_i is countably infinite?)

Demonstration problems:

4. Prove that the class of context-free languages is closed under unions, concatenations, and the Kleene star operation, i.e. if the languages $L_1, L_2 \subseteq \Sigma^*$ are context-free, then so are the languages $L_1 \cup L_2$, $L_1 L_2$ and L_1^*.

5. Prove that the class of context-free languages is not closed under intersections and complements. (*Hint:* Represent the language $\{a^k b^k c^k \mid k \geq 0\}$ as the intersection of two context-free languages.)

6. Prove that the Cartesian product $\mathbb{N} \times \mathbb{N}$ is countably infinite. (*Hint:* Think of the pairs $(m,n) \in \mathbb{N} \times \mathbb{N}$ as embedded in the Euclidean (x,y) plane $\mathbb{R}^2$. Enumerate the pairs by diagonals parallel to the line $y = -x$.) Conclude from this result that also the set $\mathbb{Q}$ of rational numbers is countably infinite.