T-79.1001/1002 **Autumn 2005**
Introduction to Theoretical Computer Science T/Y
Tutorial 5, 18–19 October
Problems

Homework problems:

1. Consider the following context-free grammars:

$$\text{(a)}\quad \begin{aligned} A &\rightarrow aAcc \mid B \\ B &\rightarrow bBc \mid \varepsilon \end{aligned} \qquad\qquad \text{(b)}\quad S \rightarrow +S- \mid SS \mid \varepsilon$$

Give a derivation for the sentence *abccc* according to grammar (a), and a derivation for the sentence $++-+--+-$ according to grammar (b). Describe the language generated by each grammar verbally as simply as you can.

2. A *palindrome* is a string w such that $w = w^R$. (E.g. "MADAMIMADAM", "ABLEWASIEREISAWELBA," cf. `http://www.palindromes.org/`.) Consider the set of palindromes over the alphabet $\{a, b\}$:

$$\text{PAL} = \{w \in \{a,b\}^* \mid w = w^R\}.$$

Design a context-free grammar generating the language. (*Hint:* Note that a string $w \in \text{PAL}$, if and only if it is of the form $w = uXu^R$, where $X = a$, b or ε.)

3. Consider the following grammar generating a certain type of list structures:

$$S \rightarrow (S) \mid S, S \mid a.$$

(a) Based on the above grammar, give a leftmost and rightmost derivation and a parse tree for the sentence "$(a, (a))$".
(b) Prove that the grammar is ambiguous.
(c) Design an unambiguous grammar generating the same language.

Demonstration problems:

4. Prove that the class of context-free languages is closed under unions, concatenations, and the Kleene star operation, i.e. if the languages $L_1, L_2 \subseteq \Sigma^*$ are context-free, then so are the languages $L_1 \cup L_2$, $L_1 L_2$ and L_1^*.

5. Design a context-free grammar describing the syntax of simple "programs" of the following form: a program consists of nested `for` loops, compound statements enclosed by `begin-end` pairs and elementary operations `a`. Thus, a "program" in this language looks something like this:

```
a;
for 3 times do
begin
   for 5 times do a;
   a; a
end.
```

For simplicity, you may assume that the loop counters are always integer constants in the range $0, \ldots, 9$.

6. (a) Prove that the following context-free grammar is ambiguous:

$$\begin{aligned} S &\rightarrow \textbf{if } b \textbf{ then } S \\ S &\rightarrow \textbf{if } b \textbf{ then } S \textbf{ else } S \\ S &\rightarrow s. \end{aligned}$$

(b) Design an unambiguous grammar that is equivalent to the grammar in item (a), i.e. that generates the same language. (*Hint:* Introduce new nonterminals B and U that generate, respectively, only "balanced" and "unbalanced" **if-then-else-**sequences.)