T-79.1001/1002 **Autumn 2005**
Introduction to Theoretical Computer Science T/Y
Tutorial 4, 11–12 October
Problems

Homework problems:

1. Give regular expressions describing the following languages:
   (a) $\{w \in \{a,b\}^* \mid w \text{ contains } abb \text{ as a substring}\}$;
   (b) $\{w \in \{a,b\}^* \mid w \text{ contains either } abb \text{ or } bba \text{ (or both) as a substring}\}$;
   (c) $\{w \in \{0,1\}^* \mid w \text{ contains exactly two 0's}\}$;
   (d) $\{w \in \{0,1\}^* \mid w \text{ contains at least two 0's}\}$;
   (e) $\{w \in \{0,1\}^* \mid w \text{ contains an even number (possibly zero) of 0's}\}$;
   (f) $\{w \in \{0,1\}^* \mid w \text{ begins and ends with different symbols}\}$;
   (g) $\{w \in \{0,1\}^* \mid |w| = 1 \pmod 3\}$.

2. (a) Construct in a systematic way (as described in your textbook) a nondeterministic finite automaton corresponding to the regular expression $((\varepsilon \cup 0)1)^*011^*$.
   (b) Make your automaton deterministic.
   (c) Describe the language in part (a) in English as simply as you can.

3. Give regular expressions describing the following languages:
   (a) $\{w \in \{a,b\}^* \mid w \text{ does not contain } abb \text{ as a substring}\}$;
   (b) $\{w \in \{0,1\}^* \mid w \text{ contains an even number of both 0's and 1's}\}$.

   (*Hint:* Design first a finite automaton for each of the languages, cf. problems 2/3(c) & 3/1, and convert these automata then in a systematic manner, as described in your textbook, into the corresponding regular expressions.)

Demonstration problems:

4. Simplify the following regular expressions (i.e., design simpler expressions describing the same languages):
   (a) $(\emptyset^* \cup a)(a^*)^*(b \cup a)b^*$
   (b) $(a \cup b)^* \cup \emptyset \cup (a \cup b)b^*a^*$
   (c) $a(b^* \cup a^*)(a^*b^*)^*$

5. Determine whether the regular expressions $r_1 = b^*a(a^*b^*)^*$ and $r_2 = (a \cup b)^*a(a \cup b)^*$ describe the same language, by constructing the minimal deterministic finite automata corresponding to them.

6. Prove that if L is a regular language, then so is $L' = \{xy \mid x \in L, y \notin L\}$.