T-79.1001/1002 **Autumn 2005**

Introduction to Theoretical Computer Science T/Y

Tutorial 1, 20–21 September

Problems

Remember to enroll for the course using the TOPI registration system by 23 September. For bookkeeping reasons, registration is **compulsory***, even if you were not intending to attend the lectures or the tutorial sessions.*

Homework problems:

1. Let $A = \{a, b, c\}$, $B = \{b, d\}$, and $C = \{a, c, d, e\}$. List the elements of the following sets:
   (a) $A \cup (C - B)$;
   (b) $B \times (A \cap C)$;
   (c) $\mathcal{P}(\{\emptyset\}) - \mathcal{P}(\emptyset)$.

2. (a) Let $A = \{a, b, c, d\}$, and define a relation $R \subseteq A \times A$ as follows:
$$R = \{(a, c), (a, d), (b, b), (c, b), (c, d), (d, b), (d, c)\}.$$
   Draw the graphs corresponding to the following relations:
$$\text{(a)}\ R, \qquad \text{(b)}\ R^{-1}, \qquad \text{(c)}\ R \circ R, \qquad \text{(d)}\ R \cap (R \circ R).$$
   Are some of these relations actually functions?
   (b) List all the equivalence relations (partitions) on the set $\{a, b, c\}$.

3. Verify by induction the correctness of the formula:
$$1 \cdot 2^1 + 2 \cdot 2^2 + \cdots + n \cdot 2^n = (n - 1) \cdot 2^{n+1} + 2.$$

Demonstration problems:

4. Let A and B be subsets of a given fundamental set U. Prove the correctness of the following *de Morgan formulas* that relate the unions, intersections, and complements of A and B to each other:
$$\overline{A \cup B} = \bar{A} \cap \bar{B}, \quad \overline{A \cap B} = \bar{A} \cup \bar{B}.$$

5. Define a relation $\sim$ on the set $\mathbb{N} \times \mathbb{N}$ by the rule:
$$(m, n) \sim (p, q) \quad \Leftrightarrow \quad m + n = p + q.$$
   Prove that this is an equivalence relation, and describe intuitively ("geometrically") the equivalence classes it determines.

6. Prove by induction that if X is a finite set of cardinality $n = |X|$, then its power set $\mathcal{P}(X)$ is of cardinality $|\mathcal{P}(X)| = 2^n$.