Answer Set Programming

an Approach to Declarative Problem Solving

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Contents

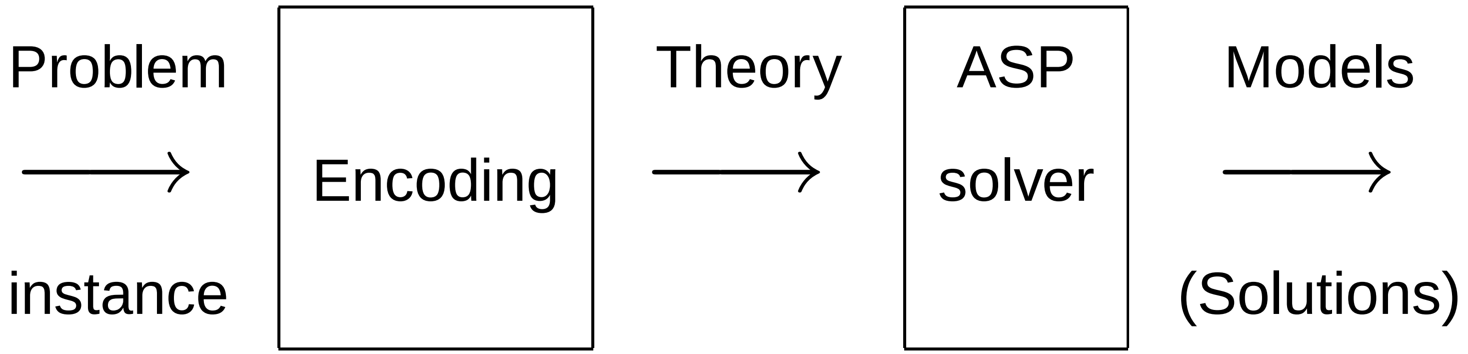
- Introduction to Answer Set Programming (ASP)
- ASP with logic programs
- Implementation techniques
- Available systems
- Applications

Answer Set Programming

- Term coined by Vladimir Lifschitz
- Roots: KR, logic programming, nonmonotonic reasoning
- Based on some formal system with semantics that assigns a theory a collection of answer sets (models).
- An **ASP solver**: computes answer sets for a theory
- Solving a problem in ASP:
  Encode the problem as a theory such that **solutions** to the problem are given by the **answer sets** of the theory.

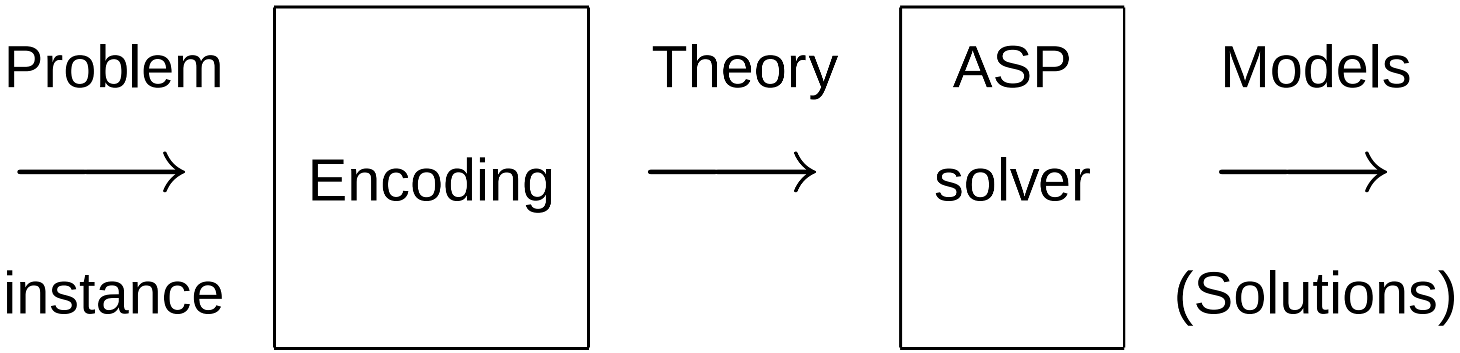
ASP—cont'd

- Solving a problem using ASP



ASP—cont'd

- Solving a problem using ASP



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Possible formal system	Models
Propositional logic	Truth assignments
CSP	Variable assignments
Logic programs	Stable models

Example. k-coloring problem

- Given a graph (V,E) find an assignment of one of k colors to each vertex such that no two adjacent vertices share a color.
- Encoding 3-coloring using propositional logic

For each vertex $v \in V$:	For each edge $(v,u) \in E$:
$v(1) \lor v(2) \lor v(3)$	$\neg v(1) \lor \neg u(1)$
$\neg v(1) \lor \neg v(2)$	$\neg v(2) \lor \neg u(2)$
$\neg v(1) \lor \neg v(3)$	$\neg v(3) \lor \neg u(3)$
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For each edge $(v,u) \in E$:

$\neg v(1) \lor \neg u(1)$
$\neg v(2) \lor \neg u(2)$
$\neg v(3) \lor \neg u(3)$

- 3-colorings of a graph (V,E) and models of the encoding correspond:
vertex v colored with color i iff $v(i)$ true in the model.

What is ASP Good for?

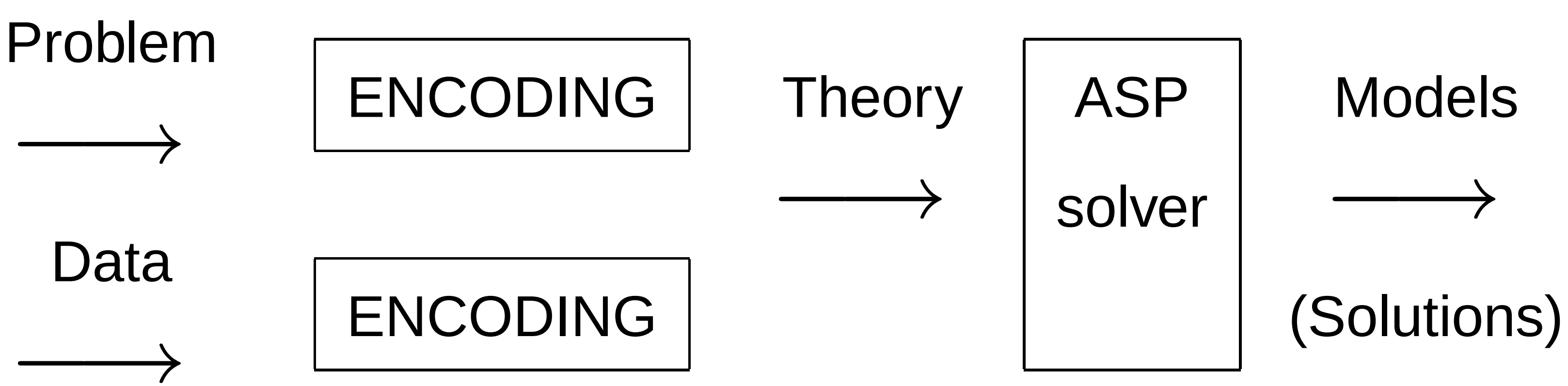
Search problems:

- Constraint satisfaction
- Planning, routing
- Computer-aided verification
- Security analysis
- Product configuration
- Combinatorics
- Diagnosis

☞ Declarative problem solving

Towards ASP in Practice

- Uniform encoding:
  separate problem specification and data
- Compact, easily maintainable representation
- Integrating KR, DB, and search techniques
- Handling dynamic, knowledge intensive applications:
  data, frame axioms, exceptions, defaults, closures



ASP Using Logic Programs

ASP Using Logic Programs

- Logic programming: framework for merging KR, DB, and search
- PROLOG style logic programming systems not directly suitable for ASP:
  - they search for proofs (not models) and produce answer substitutions
  - they are not entirely declarative
- In late 80s new semantical basis for "negation-as-failure" in LPs based on nonmonotonic logics: **Stable model semantics**
- Implementations of stable model semantics led to ASP

Example. 3-coloring

Problem:

$$clrd(V,1) \leftarrow \text{not } clrd(V,2), \text{not } clrd(V,3), vtx(V)$$
$$clrd(V,2) \leftarrow \text{not } clrd(V,1), \text{not } clrd(V,3), vtx(V)$$
$$clrd(V,3) \leftarrow \text{not } clrd(V,1), \text{not } clrd(V,2), vtx(V)$$
$$\leftarrow edge(V,U), clrd(V,C), clrd(U,C)$$

Data:

$$vtx(v) \quad vtx(u) \quad \ldots$$
$$edge(v,u) \quad edge(u,w) \quad \ldots$$

Example. 3-coloring

Problem:

$$clrd(V,1) \leftarrow \text{not}\, clrd(V,2), \text{not}\, clrd(V,3), vtx(V)$$
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$$\leftarrow edge(V,U), clrd(V,C), clrd(U,C)$$

Data:

$vtx(v)$	$vtx(u)$	$\ldots$
$edge(v,u)$	$edge(u,w)$	$\ldots$

☞ 3-colorings and stable models of the encoding correspond: v colored i iff $clrd(v,i)$ in the model.

LPs with Stable Models Semantics

- Consider normal logic program rules

$$A \leftarrow B_1, \ldots, B_m, \text{not}\, C_1, \ldots, \text{not}\, C_n$$

- Seen as constraints on an answer set (stable model):
  - if $B_1, \ldots, B_m$ are in the set and
  - none of $C_1, \ldots, C_n$ is included,

  then A must be included in the set
- A stable model is a set of atoms
  (i) which satisfies the rules and
  (ii) where each atom is **justified** by the rules.

Stable Models — cont'd

- Program:

  $b \leftarrow$

  $f \leftarrow b, \text{not}\ eb$

  $eb \leftarrow p$

Stable model:

$\{b, f\}$

Stable Models — cont'd

- Program:

  $b \leftarrow$

  $f \leftarrow b, \mathrm{not}\ eb$

  $eb \leftarrow p$

  Stable model:

  $\{b, f\}$

- Another candidate model: $\{b, eb\}$
  satisfies the rules but is not a proper stable model:
  eb is included for no reason.

Stable Models — cont'd

- Program:

  $b \leftarrow$

  $f \leftarrow b, \text{not } eb$

  $eb \leftarrow p$

  Stable model:

  $\{b, f\}$

- Another candidate model: $\{b, eb\}$
  satisfies the rules but is not a proper stable model:
  eb is included for no reason.

- Justifiability of stable models is captured by the notion of a **reduct** of a program

  ☞ The stable model semantics [Gelfond/Lifschitz,1988].

Example. Stable models

- A program can have **none**, one, or **multiple** stable models.

- Program:

$p_1 \leftarrow \text{not } q_1$

$q_1 \leftarrow \text{not } p_1$

Stable models:

$\{p_1\}$

$\{q_1\}$

- Program:

$p_1 \leftarrow \text{not } q_1$

$q_1 \leftarrow \text{not } p_1$

$\leftarrow \text{not } p_1$

$\leftarrow \text{not } q_1$

Stable models:

None

Variables

- Variables are needed for uniform encodings

  Program:

  $$clrd(V,1) \leftarrow \text{not}\, clrd(V,2), \text{not}\, clrd(V,3), vtx(V)$$
  $$clrd(V,2) \leftarrow \text{not}\, clrd(V,1), \text{not}\, clrd(V,3), vtx(V)$$
  $$clrd(V,3) \leftarrow \text{not}\, clrd(V,1), \text{not}\, clrd(V,2), vtx(V)$$
  $$\leftarrow edge(V,U), clrd(V,C), clrd(U,C)$$

  Data:

  $$\begin{array}{lll} vtx(v) & vtx(u) & \ldots \\ edge(v,u) & edge(u,w) & \ldots \end{array}$$

Variables — cont'd

- Semantics: Herbrand models
- A rule is seen as a shorthand for the set of its ground instantiations.

Example.

$$clrd(V,1) \leftarrow \text{not}\, clrd(V,2), \text{not}\, clrd(V,3), vtx(V)$$

is a shorthand for

$$clrd(v,1) \leftarrow \text{not}\, clrd(v,2), \text{not}\, clrd(v,3), vtx(v)$$
$$clrd(u,1) \leftarrow \text{not}\, clrd(u,2), \text{not}\, clrd(u,3), vtx(u)$$
$$clrd(1,1) \leftarrow \text{not}\, clrd(1,2), \text{not}\, clrd(1,3), vtx(1)$$
$$\ldots$$

Stable Models — cont'd

- A stratified program has a unique stable model (canonical model).
- It is **linear time to check** whether a set of atoms is a stable model of a ground program.
- It is **NP-complete to decide** whether a ground program has a stable model.
- Normal programs (without function symbols) give a **uniform solution** to every NP search problem.

Extensions to Normal Rules

- Encoding of choices

  choice rules: $\{a\} \leftarrow b, \text{not } c$

  disjunctive rules: $a_1 \vee a_2 \leftarrow b, \text{not } c$

- Cardinality constraints

  $2\ \{hd_1, \ldots, hd_n\}\ 4$

- Weight constraints

  $20\ [hd_1 = 6, \ldots, hd_n = 13]$

- Optimization

  $minimize\,[hd_1 = 100, \ldots, hd_n = 600]$

- Preferences, soft constraints, aggregates, . . .

Generate-and-test programming

- Basic methodology:
  - **Generator rules**: provide candidate answer sets (typically encoded using choice constructs)
  - **Tester rules**: eliminate non-valid candidates (typically encoded using integrity constraints)
  - **Optimization statements**: Criteria for preferred answer sets (typically encoded using cost functions)

Example. k-coloring problem

- k-coloring: an assignment of one of k colors to each vertex such that no two adjacent vertices share a color.
- Input: available colors and a graph
  - `color(1).,...,color(k).`
  - `vtx(v).,...`
  - `edge(v,u).,...`

k-coloring — cont'd

- An assignment of colors is represented by ground atoms of the form `clrd(v,c)` where `v` is a vertex and `c` is an available color.
- The basic idea of the encoding:
  (i) generator rules produce candidate stable models (assignments)
  (ii) tester rules eliminate candidates which do not satisfy the coloring condition.

k-coloring — cont'd

```
% Encoding of the k-coloring problem
% Generator: producing candidate stable models
1 {clrd(V,C):color(C)} 1 :- vtx(V).

% Tester: eliminate candidates
% not satisfying the coloring condition.
:- edge(V,U), color(C), clrd(V,C), clrd(U,C).
```

- Given the encoding program (the input facts and the generator and tester rules):
  k-colorings and stable models correspond.
- k-coloring: facts `clrd(v,c)` in the stable model.

Example: Review assignment

```
% DATA:
reviewer(r1). ...
paper(p1). ...
classA(r1,p1). ... % Preferred papers
classB(r1,p2). ... % Doable papers
coi(r1,p3). ...    % Conflicts of interest

% PROBLEM
% Each paper is assigned 3 reviewers
3 { assigned(P,R):reviewer(R) } 3 :- paper(P).
% No paper assigned to a reviewer with coi
:- assigned(P,R), coi(R,P).
```

Review Assignment — cont'd

```
% No reviewer has an unwanted paper.
:- paper(P), reviewer(R),
   assigned(P,R), not classA(R,P), not classB(R,P).
% No reviewer has more than 8 papers
:- 9 { assigned(P,R): paper(P) }, reviewer(R).
% Each reviewer has at least 7 papers
:- { assigned(P,R): paper(P) } 6, reviewer(R).
% No reviewer has more than 2 classB papers
:-  3 { assignedB(P1,R): paper(P1) }, reviewer(R).
assignedB(P,R) :- classB(R,P), assigned(P,R).
% Minimize the number of classB papers
minimize [ assignedB(P,R):paper(P):reviewer(R) ].
```

ASP vs Other Approaches

- SAT, CSP, (M)IP
  - Similarities: search for models (assignments to variables) satisfying a set of constraints
  - Differences: no logical variables, database, DDB or KR techniques available, search space given by variable domains
- LP, CLP:
  - Similarities: database and DDB techniques
  - Differences: Search for proofs (not models), non-declarative features

Implementing ASP Solvers

ASP Solvers

- ASP solvers need to handle two challenging tasks
  - complex data
  - search
- The approach has been to use
  - **logic programming and deductive data base techniques** for the former
  - **SAT/CSP related search techniques** for the latter
- In the current systems: separation of concerns

  ☞ A two level architecture

Architecture of ASP Solvers

Typically a two level architecture employed

- **Grounding** step handles complex data:
  - Given program P with variables, generate a set of ground instances of the rules which preserves the models.
  - LP and DDB techniques employed
- **Model search** for ground programs:
  - Special-purpose search procedures
  - Translation to SAT

SMODELS system

(`http://www.tcs.hut.fi/Software/smodels`)

program (variables) → `lparse` front-end → ground program → `smodels` search → stable models

- Front-end: (deductive) DB techniques for stratified programs
- Special purpose search engine:
  - array data structures (Dowling-Gallier type)
  - local computations for large rule sets
  - linear space requirements
  - optimization built-in

Other ASP Implementations

`dlv`	`http://www.dbai.tuwien.ac.at/proj/dlv/`
`GnT`	`http://www.tcs.hut.fi/Software/gnt/`
`CMODELS`	`http://www.cs.utexas.edu/users/tag/cmodels.html`
`ASSAT`	`http://assat.cs.ust.hk/`
`NoMoRe`	`http://www.cs.uni-potsdam.de/~linke/nomore/`
`XASP`	distributed with XSB v2.6
	`http://xsb.sourceforge.net`
`aspps`	`http://www.cs.engr.uky.edu/ai/aspps/`
`ccalc`	`http://www.cs.utexas.edu/users/tag/cc/`

Example. SOKOBAN game

- Data

```
square(1, 1).       initial_at(4,3).
square(2, 1).       initial_box(3, 4).
square(3, 1).       target_square(2, 3).
...
```

- Program

```
1 { move_to(X_2, Y_2, I) :
        same_segment(X_1, Y_1, X_2, Y_2, Dir) } 1 :-
    push(X_1, Y_1, Dir, I),
    has_neighbor(X_1, Y_1, Dir),
    time(I), I < n.
```

Applications

Applications

- Planning
  USAdvisor project at Texas Tech:
  A decision support system for the flight controllers of space shuttles
- Product configuration
  –Intelligent software configurator for Debian/Linux
  –WeCoTin project (Web Configuration Technology)
  –Spin-off (`http://www.variantum.com/`)
- Computer-aided verification
  –Partial order methods
  –Bounded model checking

Applications—cont'd

- VLSI routing
- Planning
- Combinatorial problems, network management, network security, security protocol analysis, linguistics . . .
- C. Baral. Knowledge Representation, Reasoning and Declarative Problem Solving. Cambridge University Press, 2003.
- Applying ASP
  - as a stand alone system
  - as an embedded solver

Conclusions

ASP = KR + DB + search

- ASP emerging as a viable KR tool
- Efficient implementations under development (`Smodels`, `aspps`, `dlv`, `XASP`, `CMODELS`, `ASSAT`, `...`)
- Expanding functionality and ease of use
- Growing range of applications

Topics for Further Research

- Intelligent grounding
- Model computation without full grounding
- Program transformations, optimizations
- Model search: learning, restarting, backjumping, heuristics, local search techniques
- Language extensions
- Programming methodology
- Tool support

☞ EU funded WASP working group